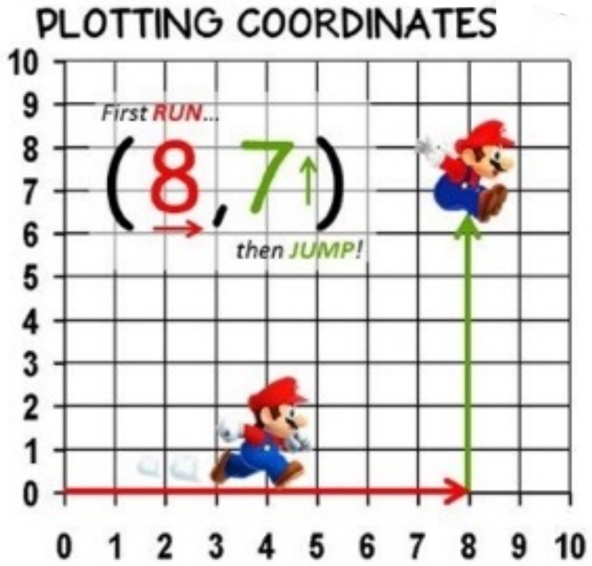
Tyler started to wish he had paid more attention in Algebra.

Straight Line Graphs



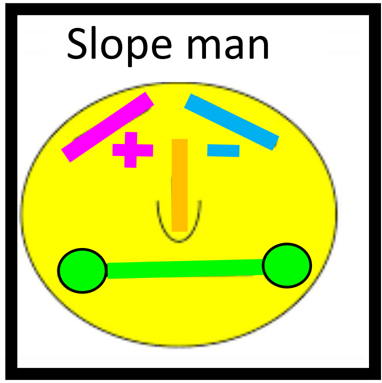
What does gradient/slope mean?

The slope/gradient is measure of how steep a line is
The slope/gradient also tells us about the direction of a line

About to die! It's impossible to climb up a vertical wall as it is far too steep!

Understanding slope means understanding two things: steepness and direction

Let’s use a skater to demonstrate this.

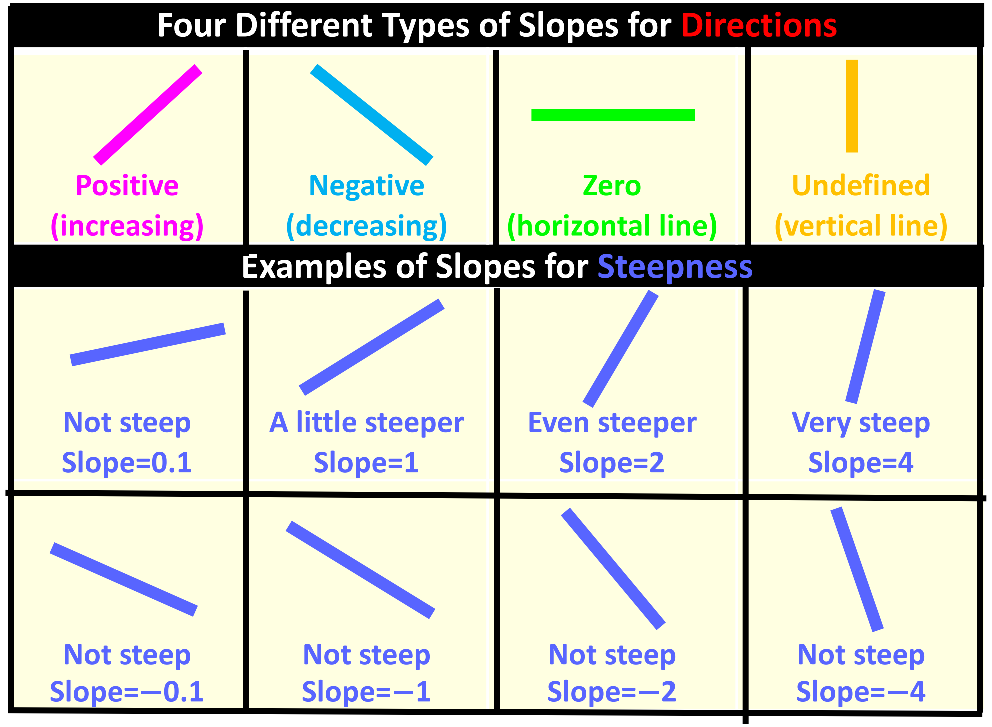


The slope of the line on the left above is **steeper** than the slope of the line on the right.
In addition, the skaters are going down the ramp from the left to the right. This means the slope **decreasing,** or negative.

How about if the skaters were going up the ramp? This would mean that the slope is **Increasing,** or positive.

So, slope measures the **direction** of the line – whether or not the skater is going up the ramp (positive slope) or going down the ramp (negative slope). It also measures the **steepness** of a line - the steeper the ramp the larger the value will be for the slope.

A skater doesn’t always skate on an incline though. A skater could also skate on flat ground, which would mean that there is no steepness to the line and therefore it would be defined as **zero slope.** Or what about if the skater was a show-off and wanted to go straight down the side of a building or ramp? This is known as an **undefined slope**.



We will see how to find the numbers for the slope over the next few pages

Let's look at our four different types of lines in a bit more detail

Line 2

end here

This is a positive slope
The man is walking UP the hill

rise

run

start here

slope $= \dfrac{\text{how much}\uparrow}{\text{how much}\rightarrow} = \dfrac{rise}{run}$

(the slope is positive since it increases from LEFT to RIGHT)

start here

Line 3

This is a negative slope
The man is running DOWN the hill

rise

run

end here

Important: This rise will be a negative answer, since it is a negative rise i.e. a fall)

slope $= \dfrac{\text{how much}\downarrow}{\text{how much}\rightarrow} = \dfrac{rise}{run}$

(the slope is negative since it decreases from LEFT to RIGHT)

Line 1

the slope is undefined (the man can't walk up that line)

Line 4

the slope is zero (the man is walking on flat ground)

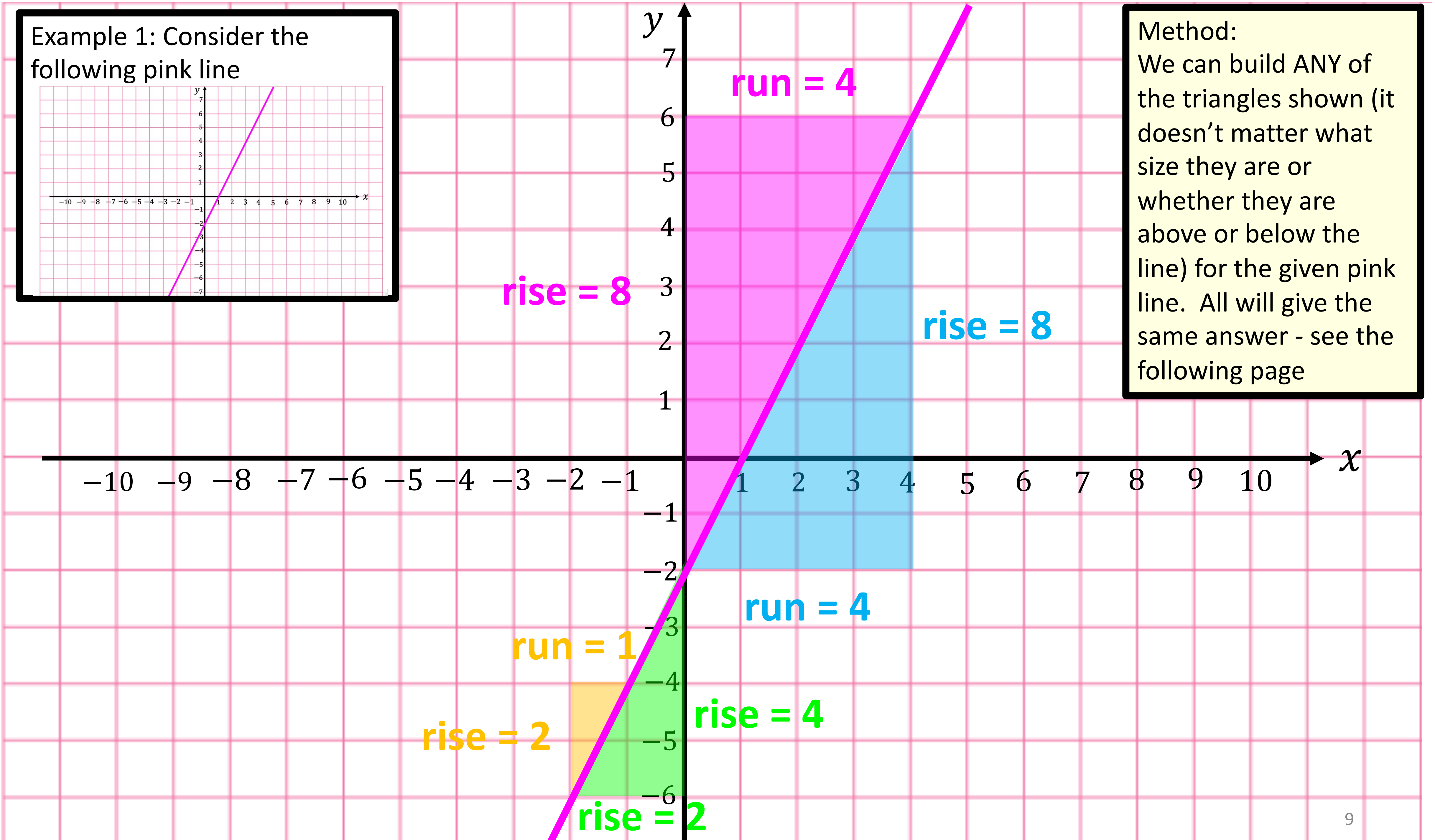
Note: $\dfrac{\text{rise}}{\text{run}}$ is just the same as $\dfrac{\text{change in } y}{\text{change in } x}$

We will see on the following pages how to get the actual value of the slope

How do we calculate the gradient/slope?

Way 1:
From A Graph-
Build A Triangle

Example 1: Consider the following pink line

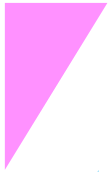


Method:
We can build ANY of the triangles shown (it doesn't matter what size they are or whether they are above or below the line) for the given pink line. All will give the same answer - see the following page

Calculating the value of the slope/gradient

As mentioned, we can build any triangle to find the slope/gradient. It doesn't matter where we form it above or below the line. We use the letter m to represent slope. Carrying on from example 1 above:

The formula for slope is $\text{slope} = m = \dfrac{rise}{run}$

Using the pink triangle  $: \ \mathbf{m} = \dfrac{rise}{run} = \dfrac{\mathbf{8}}{\mathbf{4}} = \mathbf{2}$

Using the blue triangle. $: \ \mathbf{m} = \dfrac{rise}{run} = \dfrac{\mathbf{8}}{\mathbf{4}} = \mathbf{2}$

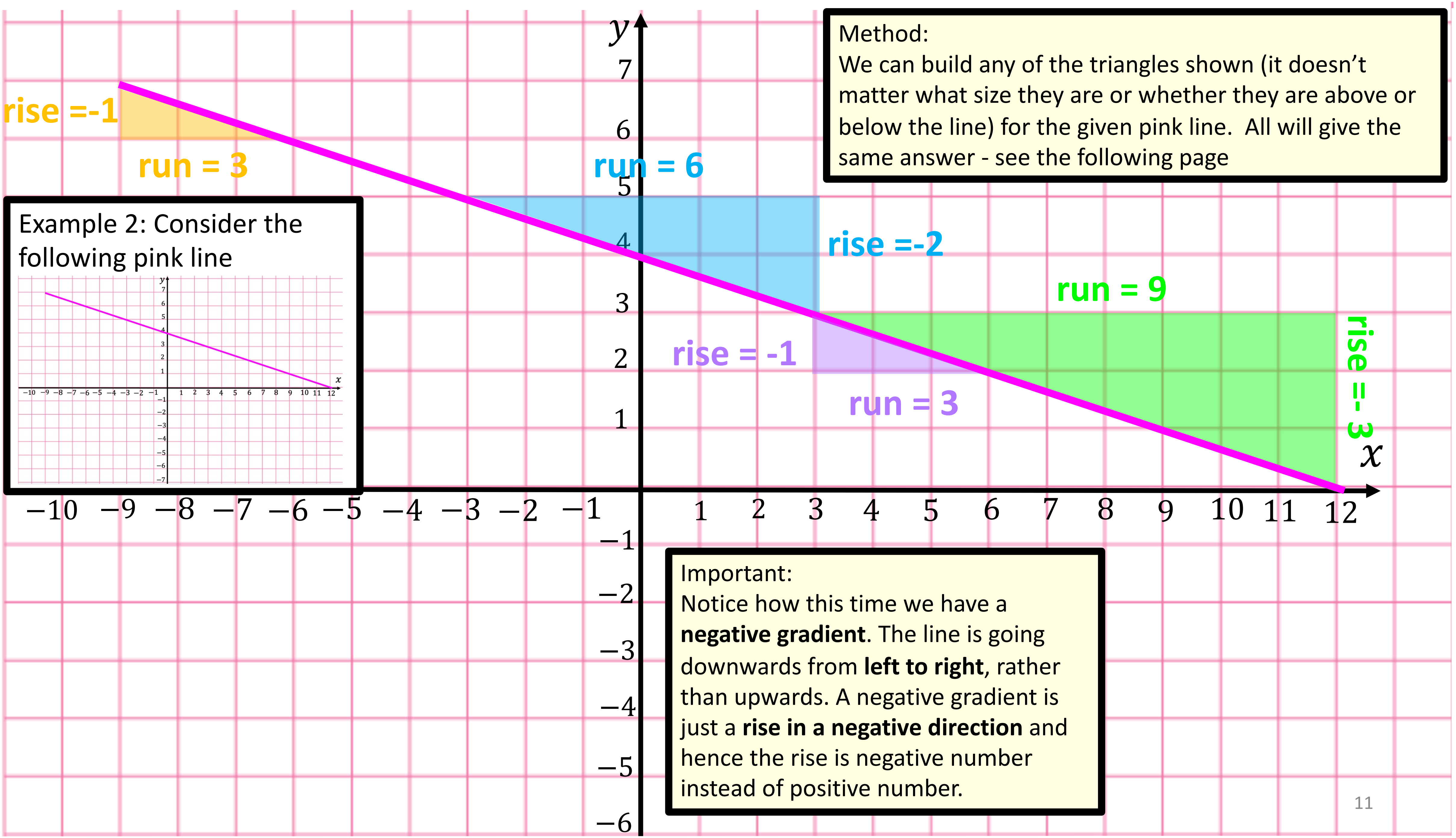
Using the orange triangle. $: \ \mathbf{m} = \dfrac{rise}{run} = \dfrac{\mathbf{2}}{\mathbf{1}} = \mathbf{2}$

Using the green triangle. $: \ \mathbf{m} = \dfrac{rise}{run} = \dfrac{\mathbf{4}}{\mathbf{2}} = \mathbf{2}$

Notice how all give the same answer for the slope which is 2. Some just need to be simplified in order to see that they give the same value!

$$\text{slope} = m = 2$$

Example 2: Consider the following pink line



Method:
We can build any of the triangles shown (it doesn't matter what size they are or whether they are above or below the line) for the given pink line. All will give the same answer - see the following page

Important:
Notice how this time we have a **negative gradient**. The line is going downwards from **left to right**, rather than upwards. A negative gradient is just a **rise in a negative direction** and hence the rise is negative number instead of positive number.

Calculating the value of the slope/gradient

$$\text{slope} = m = \frac{rise}{run}$$

Note: our rise is negative since we fall this time (negative rise)

Using the blue triangle $: \mathbf{m} = \frac{rise}{run} = \frac{-2}{6} = -\frac{1}{3}$

Using the orange triangle $: \mathrm{m} = \frac{rise}{run} = \frac{-1}{3}$

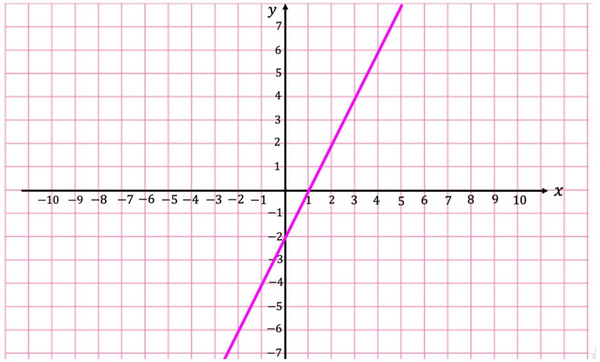
Using the green triangle $: \mathbf{m} = \frac{rise}{run} = \frac{-3}{9} = -\frac{1}{3}$

Using the purple triangle $: \ \mathbf{m} = \frac{rise}{run} = \frac{-1}{3}$

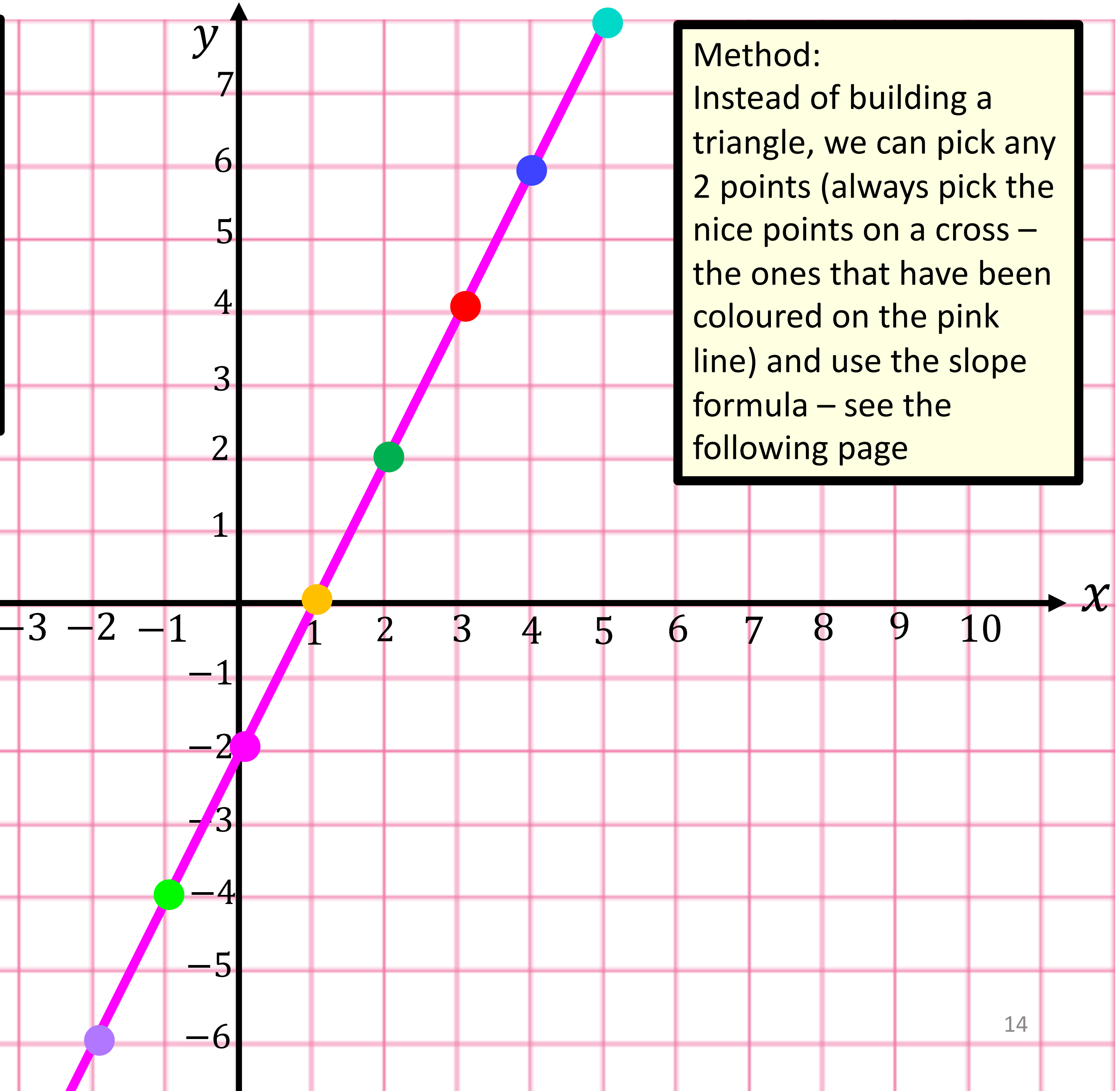
$$\text{slope} = m = -\frac{1}{3}$$

Way 2 : From A Graph - Pick <u>Any</u> Two Points On A Line

Example 3: Consider the same pink line as in example 1



Method:
Instead of building a triangle, we can pick any 2 points (always pick the nice points on a cross – the ones that have been coloured on the pink line) and use the slope formula – see the following page



Let's first learn what the formula for the slope is when we have two points. Let's generally call the points (x_1, y_1) and (x_2, y_2)

Way 1 (left to right)

subtract subtract

$(x_1, y_1),(x_2, y_2)$

Formula

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

Way 2 (right to left)

subtract subtract

$(x_1, y_1),(x_2, y_2)$

Formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Method:

This formula basically says:
we subtract the y coordinates and divide by the answer we get by subtracting the x coordinates

$$\frac{y_2 - y_1}{x_2 - x_1} \; or \; \frac{y_2 - y_1}{x_1 - x_2}$$

The formula should make sense because

$\frac{\text{rise}}{\text{run}} = \frac{\updownarrow}{\leftrightarrow}$ which is just $\frac{\text{change in y}}{\text{change in x}}$

Why are there 2 ways? It doesn't matter which way round we subtract, as long as we stay consistent!

So, for our graph for example 3 on the previous page, we had the following coordinates

● $(5,8)$ ● $(4,6)$ ● $(3,4)$ ● $(2,2)$ ● $(1,0)$ ● $(0,-2)$ ● $(-1,-4)$ ● $(-2,-6)$

Pick ANY pair of coordinates. Let's choose $(5,8)$ and $(0,-2)$

Way 1

$$m = \frac{8 - -2}{5 - 0} = \frac{8+2}{5} = 2$$

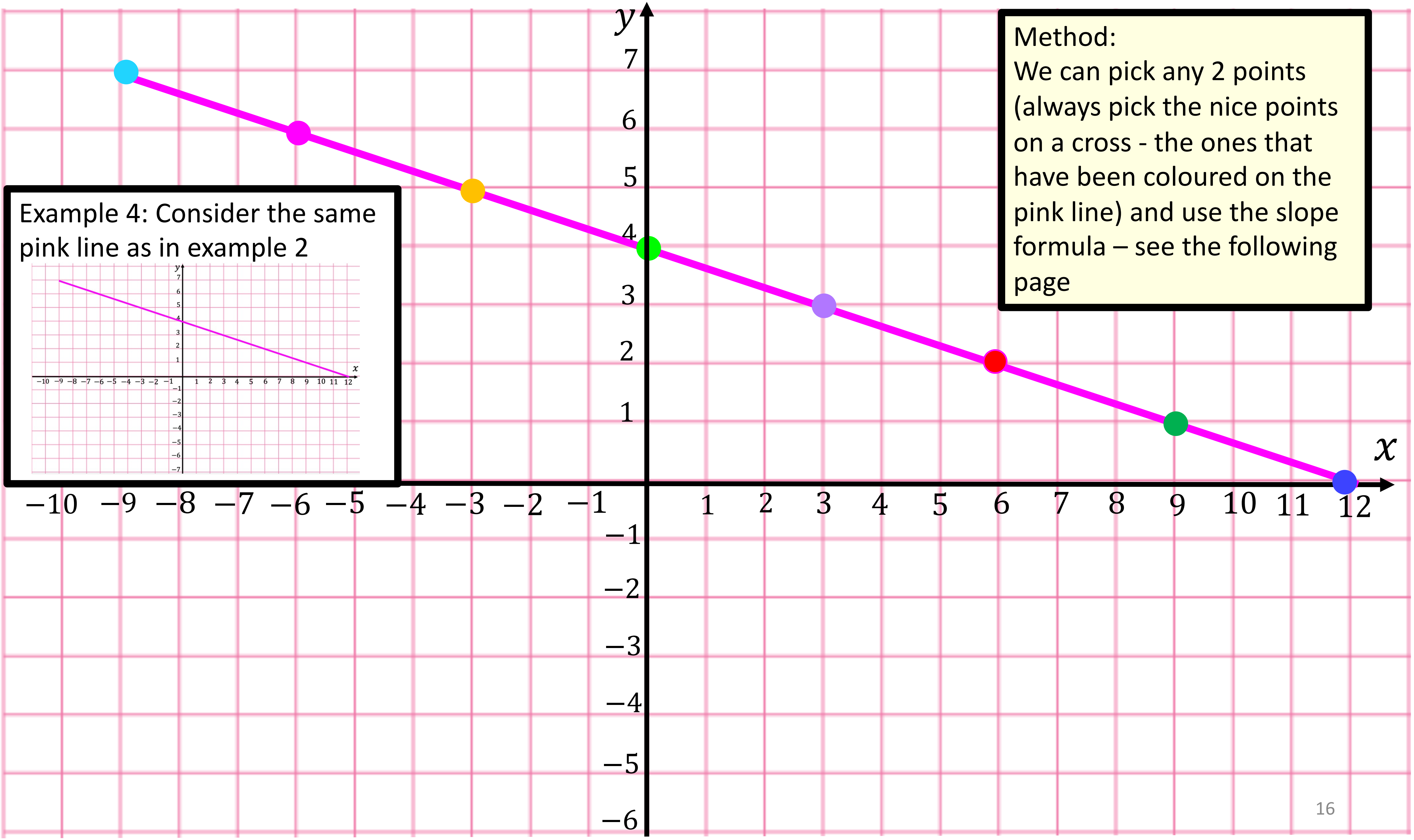
Way 2

$$m = \frac{-2 - 8}{0 - 5} = \frac{-10}{-5} = 2$$

slope = $m = 2$

Note: picking any two coordinates would have still given us the same answer

Example 4: Consider the same pink line as in example 2



Method:
We can pick any 2 points (always pick the nice points on a cross - the ones that have been coloured on the pink line) and use the slope formula – see the following page

Recall the slope formula:

Way 1 (left to right)

subtract subtract

$$(x_1, y_1),(x_2, y_2)$$

Formula

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

Way 2 (right to left)

subtract subtract

$$(x_1, y_1),(x_2, y_2)$$

Formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

So, for our graph for example 4 on the previous page, we had the coordinates

$(-9,7)$ $(-6,6)$ $(-3,5)$ $(0,4)$ $(3,3)$ $(6,2)$ $(9,1)$ $(12,0)$

Pick ANY pair of coordinates. Let's choose $(-3,5)$ and $(3,3)$

Way 1

$$m = \frac{5-3}{-3-3} = \frac{2}{-6} = -\frac{1}{3}$$

Way 2

$$m = \frac{3-5}{3--3} = \frac{-2}{6} = -\frac{1}{3}$$

$$\text{slope} = m = -\frac{1}{3}$$

Way 3 : From A Table Of Values

When given a table of values, we could just plot all the points, form the line and then use the previous methods learnt such as building triangles or picking two points on the line, but we don't have to! There is a quicker way when we're given table of values!

x	-3	-2	-1	0	1	2	3
y	-11	-8	-5	-2	1	4	7

$+3$ $+3$ $+3$ $+3$ $+3$ $+3$

The slope is just the constant number that y is changing by. Here we keep adding 3, so the slope is 3

$$\text{slope} = m = 3$$

Note: This only works because the x values are changing by one each time in the table. If the table only consisted of even values for x say, then we would get twice the slope.

Sometimes we'll be a given a table and sometimes we'll need to build it. We will see how to do this later on in the how to graph a line section.

Way 4 : From Two Coordinates

We have already seen how to do this when we dealt with choosing two coordinates on the line! Here we are just already given the coordinates for free and don't have to pick them off the graph!

Way 1

subtract subtract

$$(x_1, y_1)(x_2, y_2)$$

$$\text{slope} = m = \frac{y_1 - y_2}{x_1 - x_2}$$

Way 2

subtract subtract

$$(x_1, y_1)(x_2, y_2)$$

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Remember, it doesn't matter which way round we subtract, as long as we stay consistent (hence we have 2 ways)

For example: Find slope of the line passing through the points $(-1,2)$ and $(4,-5)$

$$\frac{2--5}{-1-4} = \frac{7}{-5} = -\frac{7}{5} \quad \text{or} \quad \frac{-5-2}{4--1} = \frac{-7}{5} = -\frac{7}{5}$$

Way 5 : From The Equation Of A Line

The equation of a line looks like $y = mx + c$

There are 2 values that are important: m and c. We have already seen that m represents the slope

$$y = mx + c$$

The gradient/slope is just this value of m right here in front of x

Note: We will see what the c value means in a bit

Let's look at some examples

$y = x - 2$	$y = 2x - 1$	$y = -x + 4$	$y = -2 + 3x$	$y = 2 - 4x$	$x = 4$	$y = 5$
$y = x + 2$ means $y = 1x + 2$ gradient $= 1$	gradient $= 2$	$y = x + 2$ means $y = -1x + 4$ gradient $= -1$	Need to re-order this first $y = 3x - 2$ gradient $= 3$	Need to re-order this first $y = -4x + 2$ gradient $= -4$	This is a vertical line since x is the same value the whole time. The gradient here is undefined	This is a horizontal line since y is the same value the whole time. $y = 5$ is like writing $y = 0x + 5$ gradient $= 0$

$y + x = 4$	$y - 2x = 5$	$2x + 4y = 5$	$5x - 2y = 7$
We need to use algebra to re-arrange $y = -x + 4$ gradient $= -1$	We need to use algebra to re-arrange $y = 2x + 5$ gradient $= 2$	We need to use algebra to re-arrange $4y = -2x + 5$ $y = \frac{-2x + 5}{4}$ $y = -\frac{1}{2} + \frac{5}{4}$ gradient $= -\frac{1}{2}$	We need to use algebra to re-arrange $-2y = -5x + 7$ $y = \frac{-5x + 7}{-2}$ $y = \frac{5}{2}x - \frac{7}{2}$ gradient $= \frac{5}{2}$

What is the y intercept and how do we find it?

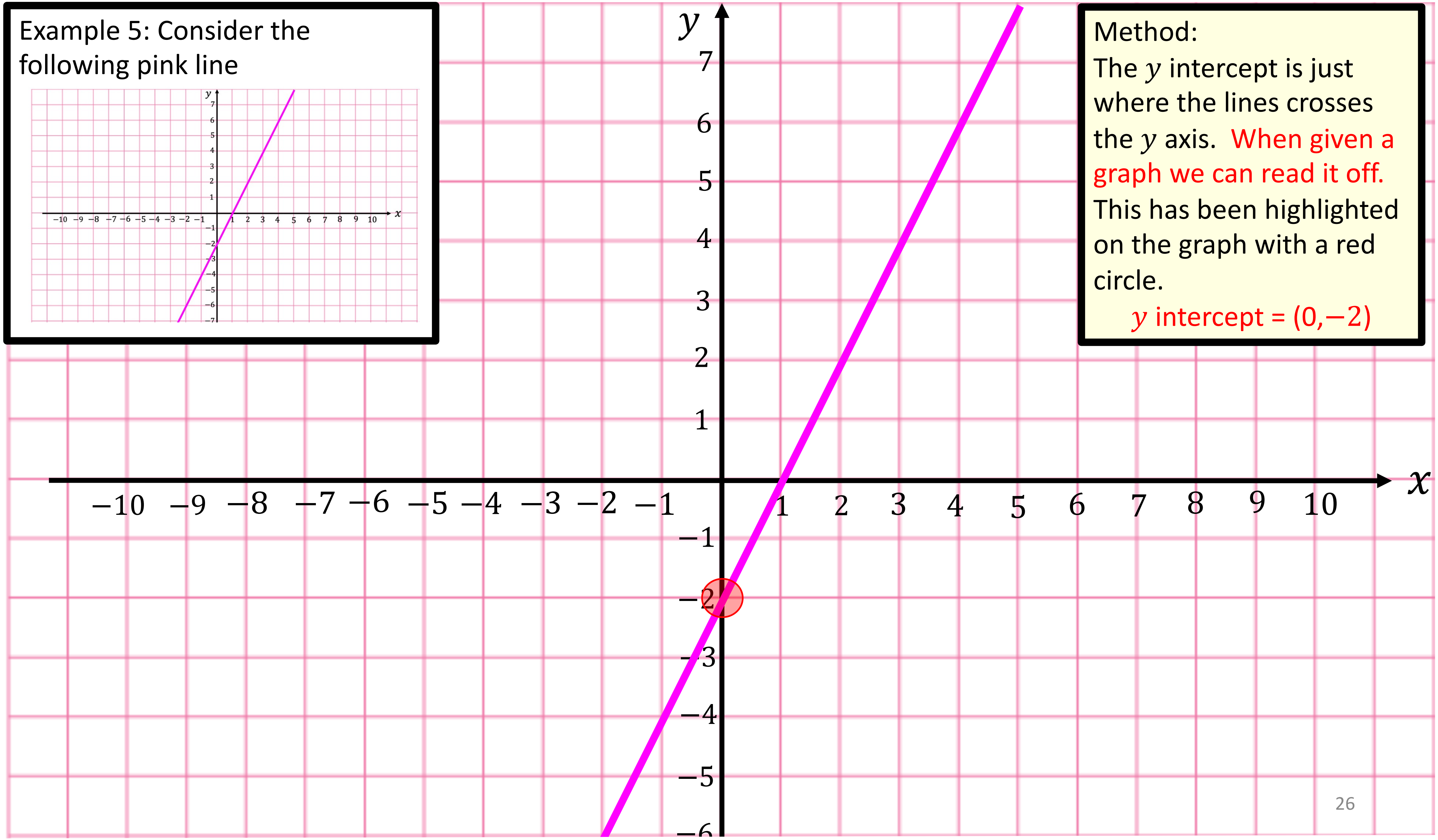
Way 1:
From A Graph

Example 5: Consider the following pink line

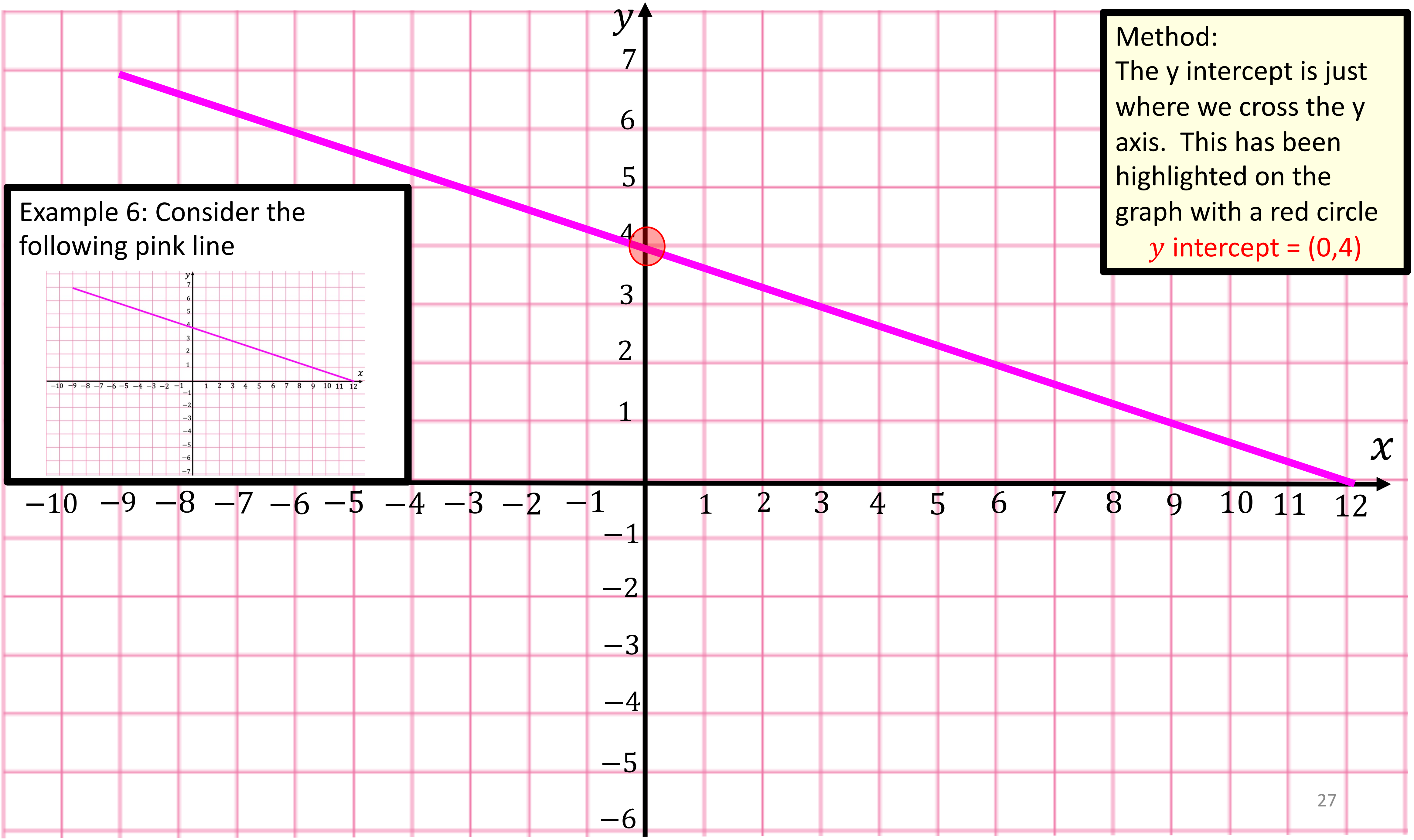
Method:

The y intercept is just where the lines crosses the y axis. When given a graph we can read it off. This has been highlighted on the graph with a red circle.

y intercept = $(0,-2)$



Example 6: Consider the following pink line



Method:
The y intercept is just where we cross the y axis. This has been highlighted on the graph with a red circle

y intercept = (0,4)

Way 2:
From An Equation

The equation of a line looks like $y = mx + c$

The y intercept is represents by the letter c

$$y = mx + c$$

The y intercept is this value here. We use the letter c to represent the y intercept.
Note: some courses use the letter b instead of c to represent the slope

Let's look at some examples

$y = 2x - 1$	$y = -x + 4$	$y = -2 + 3x$	$y = 2 - 4x$	$x = 4$	$y = 5$
y intercept is 4 $(0,4)$	Need to re-order this first $y = 3x - 2$ y intercept is -2 $(0,-2)$	Need to re-order this first $y = -4x + 2$ y intercept is 2 $(0,2)$	This is a vertical line since x is the same value the whole time. There is no y intercept	This is a horizontal line since y is the same value the whole time. $y = 5$ is like writing $y = 0x + 5$ y intercept is 5 $(0,5)$	y intercept is -1 $(0,-1)$

$y + x = 4$	$y - 2x = 5$	$2x + 4y = 5$	$5x - 2y = 7$
We need to use algebra to re-arrange $y = -x + 4$ y intercept is 4 $(0,4)$	We need to use algebra to re-arrange $y = 2x + 5$ y intercept is 5 $(0,5)$	We need to use algebra to re-arrange $4y = -2x + 5$ $y = \frac{-2x + 5}{4}$ $y = -\frac{1}{2} + \frac{5}{4}$ y intercept is $\left(0, \frac{5}{4}\right)$	We need to use algebra to re-arrange $-2y = -5x + 7$ $y = \frac{-5x + 7}{-2}$ $y = \frac{5}{2}x - \frac{7}{2}$ y intercept is $\left(0, -\frac{7}{2}\right)$

How do we graph an equation of a line?

Way 1:
Build A Table Of Values

Graph the line $y = 2x - 1$

Pick x values, let's say -3 to 3 (you are normally given the table with x values already chosen, but if not choose your own and draw out the following table)

x	-3	-2	-1	0	1	2	3
y							

Plug in the x values into the equation $y = 2x - 1$ in order find the y values

x	-3	-2	-1	0	1	2	3
y	$2(-3) - 1$	$2(-2) - 1$	$2(-1) - 1$	$2(0) - 1$	$2(1) - 1$	$2(2) - 1$	$2(3) - 1$

Simplify each y

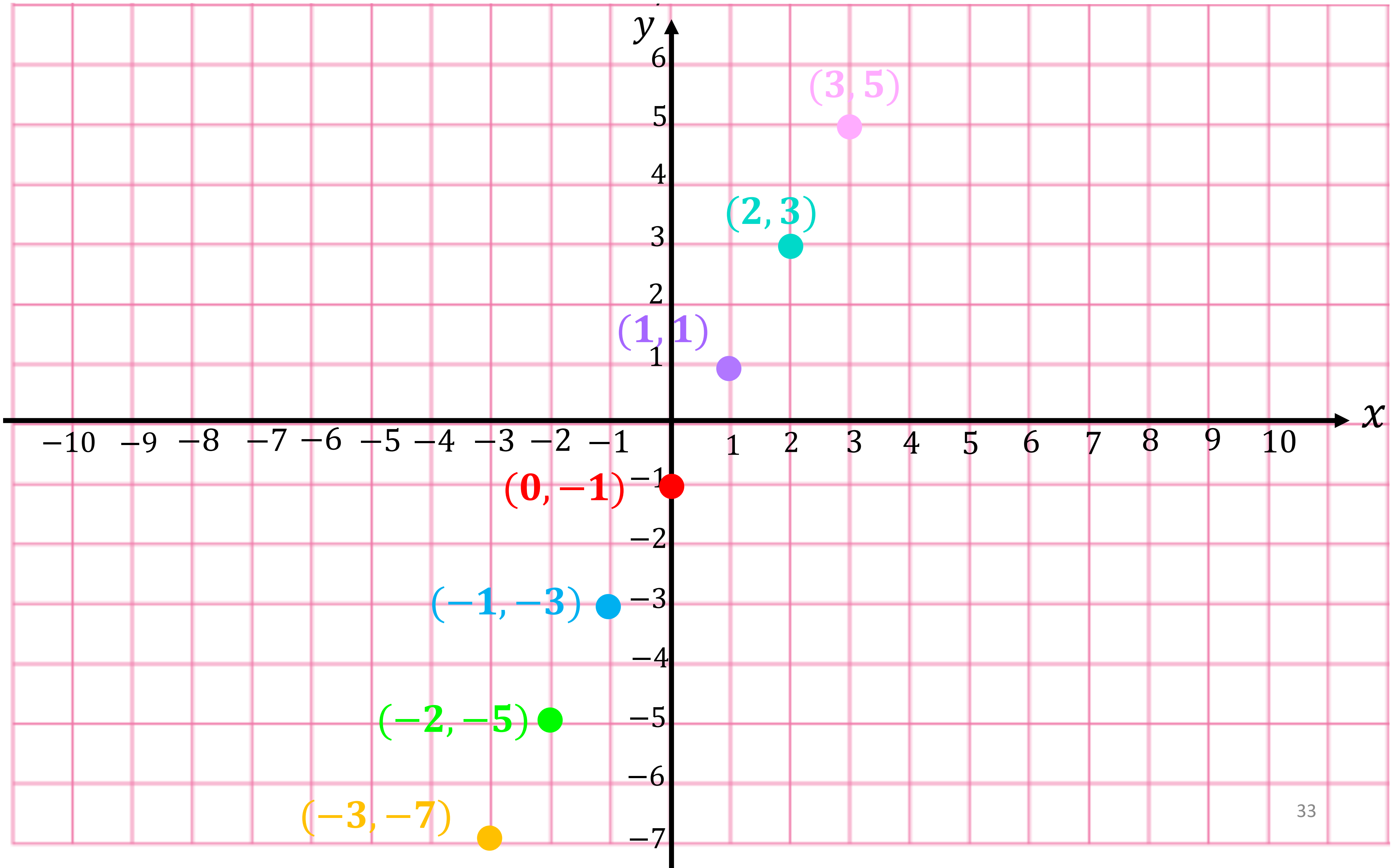
x	-3	-2	-1	0	1	2	3
y	-7	-5	-3	-1	1	3	5

Let's colour code each coordinates

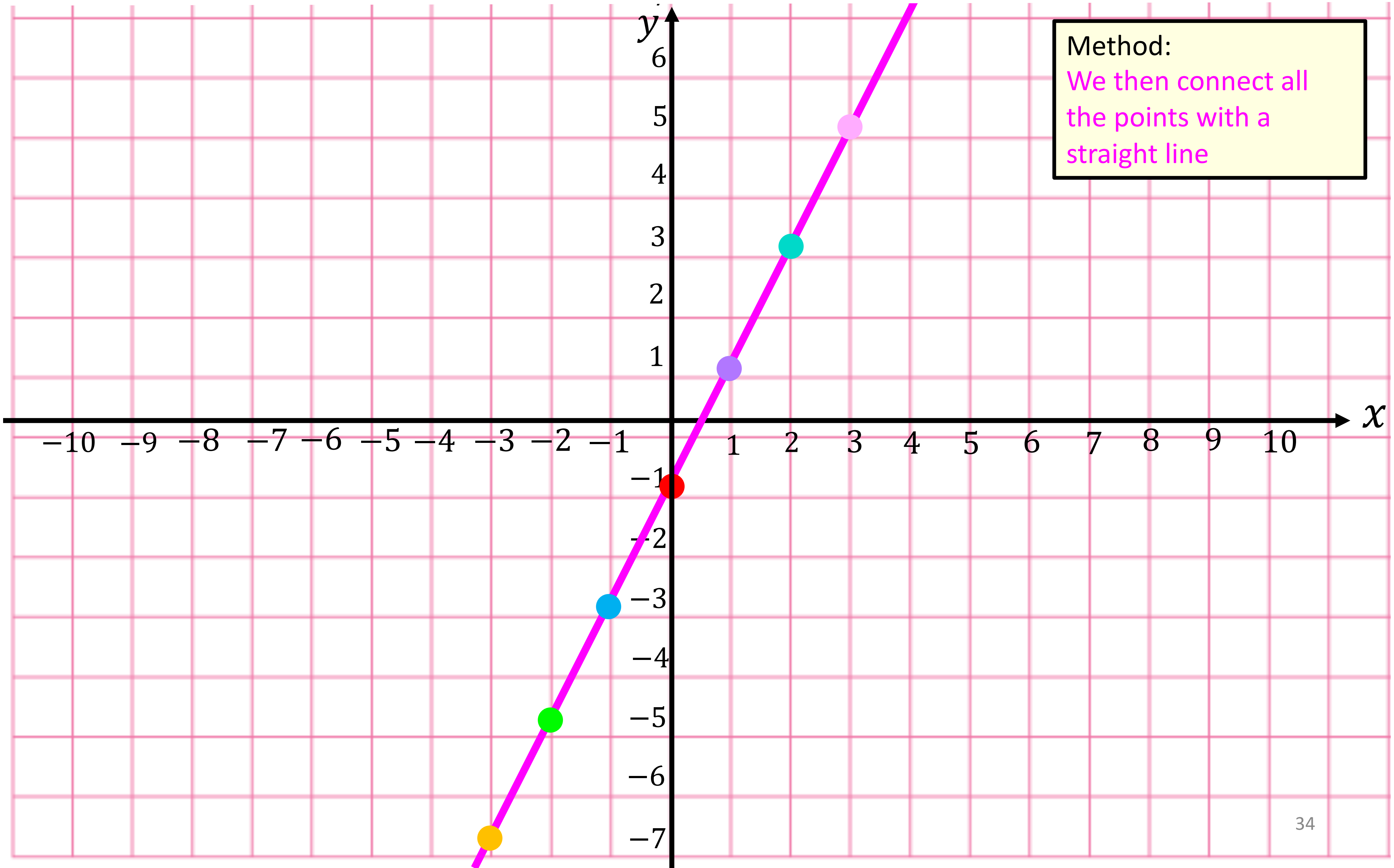
x	-3	-2	-1	0	1	2	3
y	-7	-5	-3	-1	1	3	5

Note: Harder questions don't always give the line in the form $y = mx + c$. We need to use algebra to make y the subject in order to get into the form $y = mx + c$ first before building the table of values.

Plot each pair of points (each colour pair). We will do this on the next page



y
x
6
5
4
3
2
1
−1
−2
−3
−4
−5
−6
−7
−10 −9 −8 −7 −6 −5 −4 −3 −2 −1
1 2 3 4 5 6 7 8 9 10
$(3, 5)$
$(2, 3)$
$(1, 1)$
$(0, -1)$
$(-1, -3)$
$(-2, -5)$
$(-3, -7)$



Method:
We then connect all the points with a straight line

Way 2:

Start with the y intercept and move by the gradient

Before we start, the following can help to remember what we are about to learn :

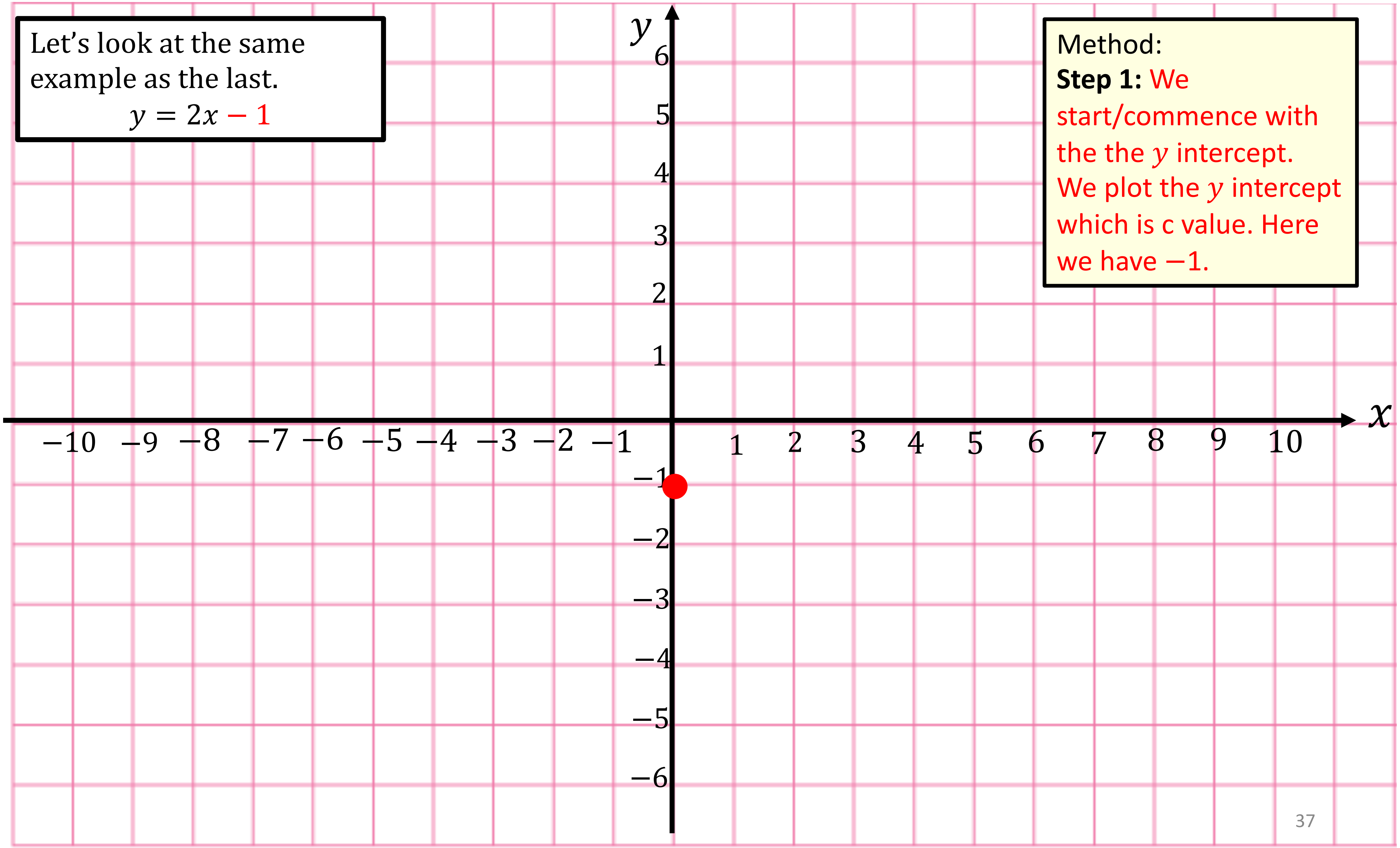
$$y = mx + c$$

M	C
O	O
V	M
E	M
	E
	N
	C
	E

The c value is where we **commence** and the value m is **how we move** to the next point on the graph

Let's look at the same example as the last.

$y = 2x - 1$

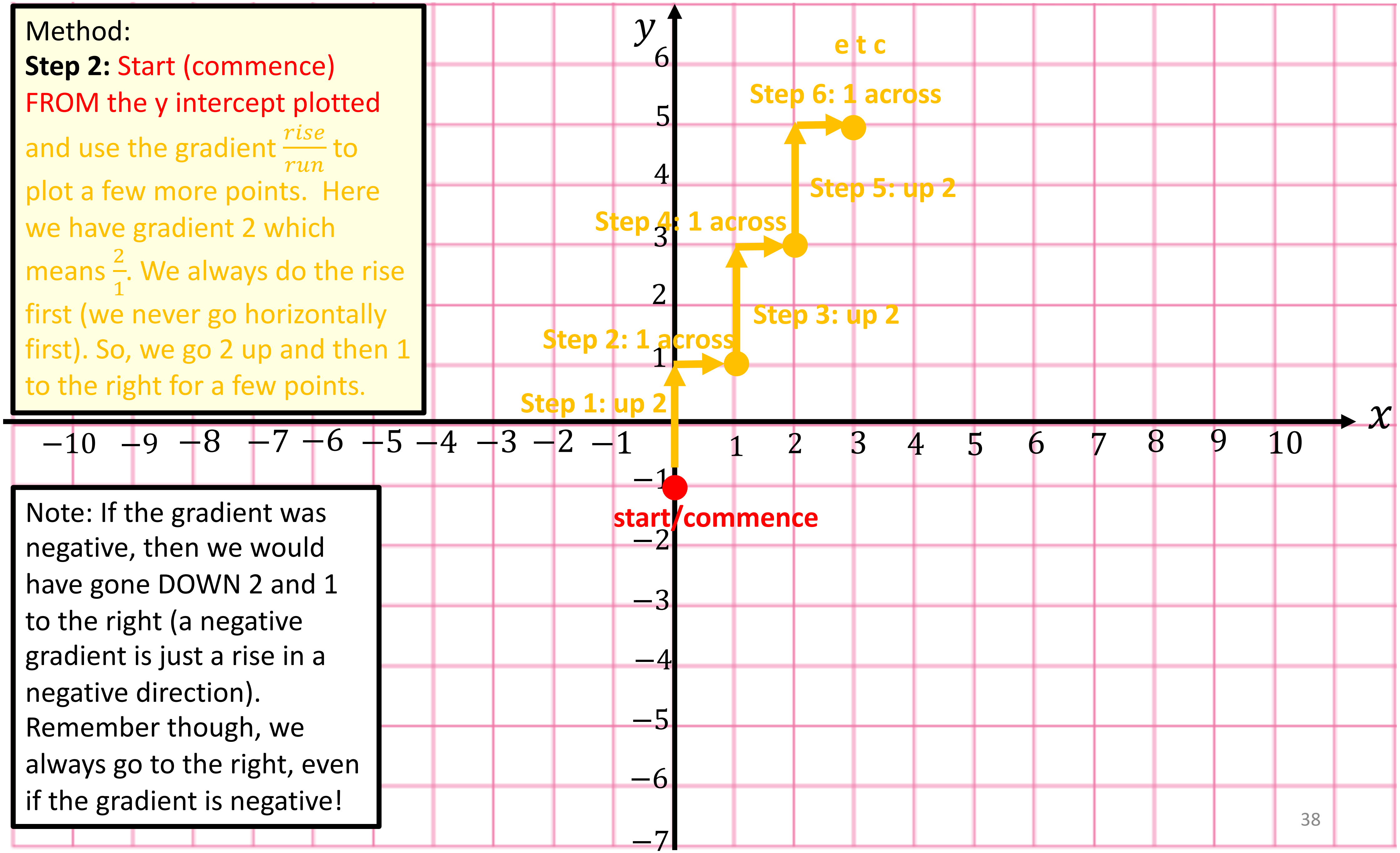


Method:

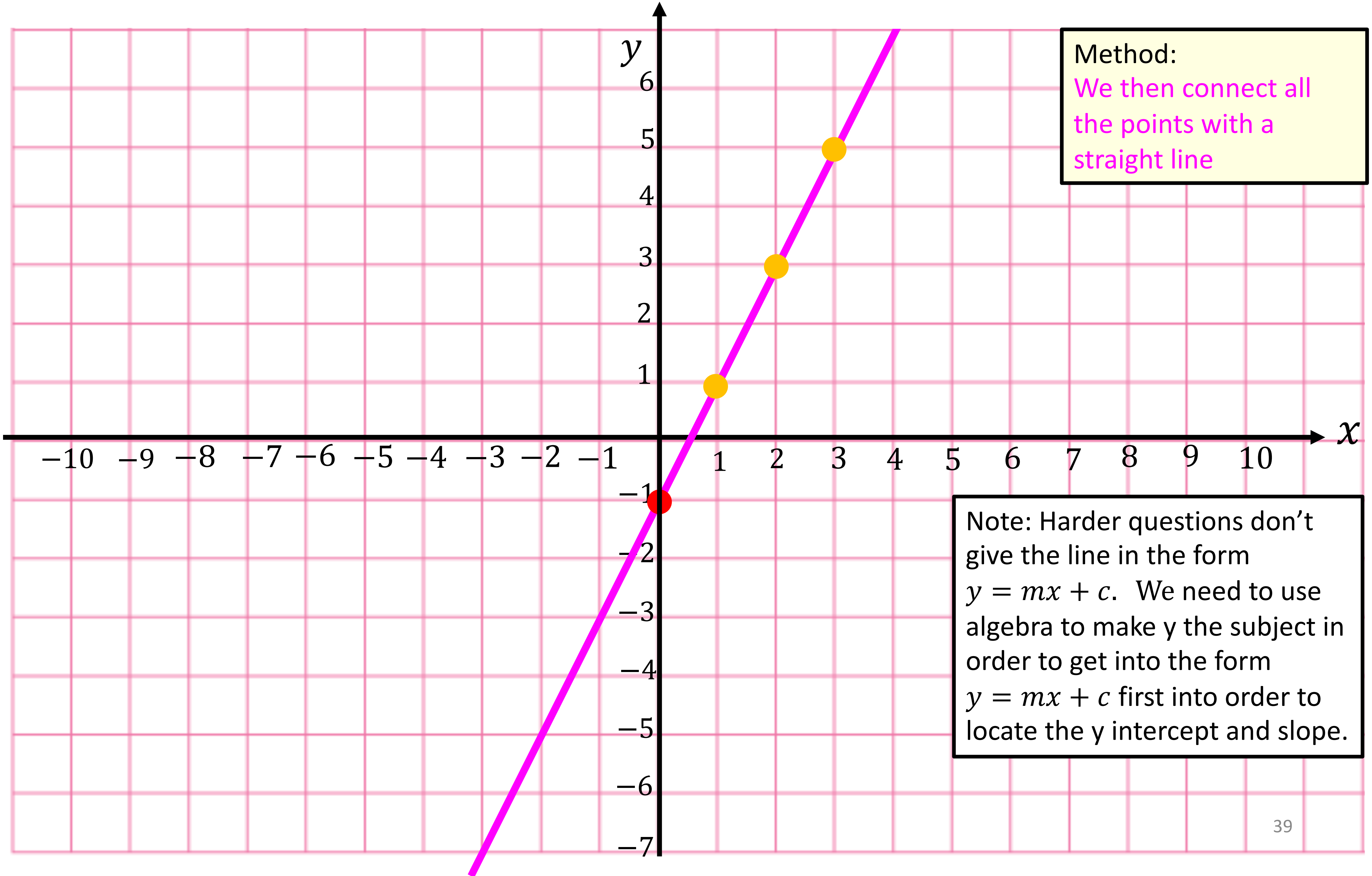
Step 1: We start/commence with the the y intercept. We plot the y intercept which is c value. Here we have -1.

Method:

Step 2: Start (commence) FROM the y intercept plotted and use the gradient $\frac{rise}{run}$ to plot a few more points. Here we have gradient 2 which means $\frac{2}{1}$. We always do the rise first (we never go horizontally first). So, we go 2 up and then 1 to the right for a few points.



Note: If the gradient was negative, then we would have gone DOWN 2 and 1 to the right (a negative gradient is just a rise in a negative direction). Remember though, we always go to the right, even if the gradient is negative!



Method:
We then connect all the points with a straight line

Note: Harder questions don't give the line in the form $y = mx + c$. We need to use algebra to make y the subject in order to get into the form $y = mx + c$ first into order to locate the y intercept and slope.

Way 3:

Find two coordinates, plot them and "connect the dots"

A line is defined by two points. If we have two points, then we can connect the points just like "connecting the dots" and create the line. What points should be pick? Zero is a really easy number isn't it, so let's try $x = 0$ and $y = 0$.

For example, graph the line $y = 2x - 6$

Let $x = 0$

$x = 0$ means we replace x with 0 in the equation $y = 2x - 6$

$$y = 2(0) - 6$$

We now need to solve for y. This is easy since y is already on its own

$$y = 0 - 6$$

$$y = -6$$

So, we have the point $(0, -6)$

Let $y = 0$

$y = 0$ means we replace y with 0 in the equation $y = 2x - 6$

$$0 = 2x - 6$$

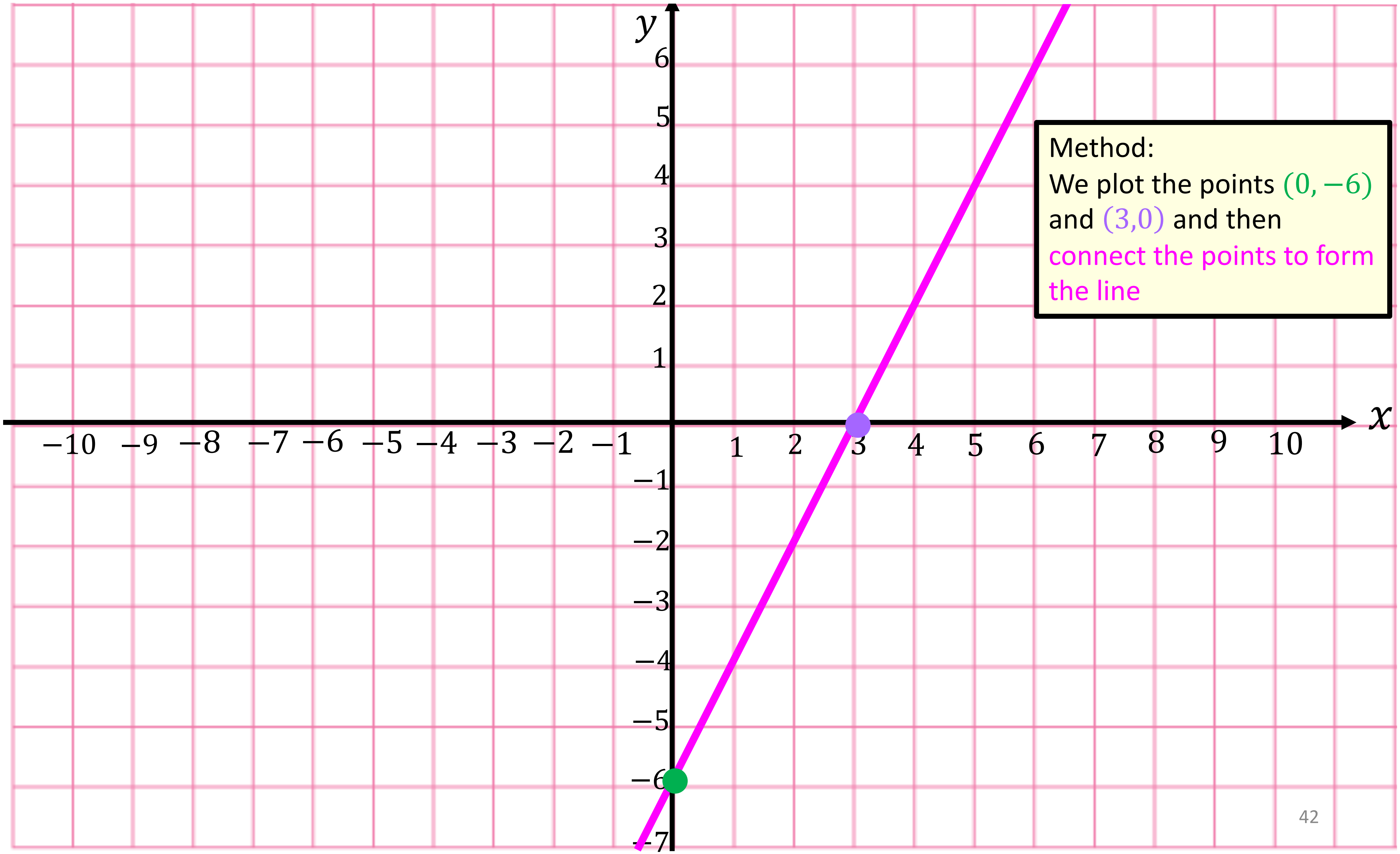
We now need to solve for x. This time we need to re-arrange to find x using algebra as it is not already on its own

$$2x = 6$$

$$x = 3$$

So, we have the point $(3,0)$

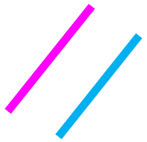
$(0, -6)$ and $(3,0)$ give us two points that define the line. To graph the line, let's now plots the 2 points and connect them.



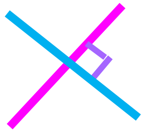
Method:
We plot the points $(0, -6)$ and $(3,0)$ and then connect the points to form the line

What are parallel and perpendicular lines?

Parallel lines the lines have the **same** gradient . They never meet
For example, if one line has a slope of 2 then a parallel line will also have a slope of 2.



Perpendicular lines meet at right angles. This means the slopes multiply to make -1 or they are negative reciprocals of each other. The easiest way to find the **negative reciprocal** is to simply flip the fraction and change the sign (a positive gets changed to a negative and a negative gets changed to a positive). For example, if one line has a slope of 2 then a perpendicular line will have a slope of $-\frac{1}{2}$.



Notice how the signs of the gradient change since one gradient will be positive and one will be negative

Let's look at some examples for perpendicular lines as this is a hard concept for some students.

- If a line has slope 2, what slope would a perpendicular line have?
  slope 2 means the same thing as $\frac{2}{1}$. Flipping the fraction gives $\frac{1}{2}$. Changing the sign means we have a negative, so $-\frac{1}{2}$. Hence a perpendicular line has slope $-\frac{1}{2}$.
  Let's check if we have done this correctly by checking if the slopes multiply to make -1:
  $$2\left(-\frac{1}{2}\right) = -1.\ \text{Yes, they do, as we expected!}$$
- If a line has slope $-\frac{2}{3}$, what slope would a perpendicular line have?
  Flipping the fraction gives $\frac{3}{2}$. Changing the sign means we have a positive. Hence a perpendicular line has slope $\frac{3}{2}$.
  Let's check if we have done this correctly by checking if the slopes multiply to make -1:
  $$-\frac{2}{3}\left(\frac{3}{2}\right) = -1.\ \text{Yes, correct again!}$$
- If a line has slope $\frac{1}{3}$, what slope would a perpendicular line have?
  Flipping the fraction gives $\frac{3}{1}$. Changing the sign means we have a negative so $-\frac{3}{1}$. Hence a perpendicular line has slope $-\frac{3}{1}$ which is just -3.
  Let's check if we have done this correctly by checking if the slopes multiply to make -1:
  $$\frac{1}{3}(-3) = -1.\ \text{Yes, correct again!}$$

How do we find the equation of a line?

The equation of a straight line looks like

$$y = mx + c$$

Recall that we use the letter m for gradient/slope and the letter c for y intercept

$$\underbrace{y = m}_{\text{gradient/slope}}x + \underbrace{c}_{\text{y intercept}}$$

So, we just need to find the gradient/slope m and y intercept c and then we are done!

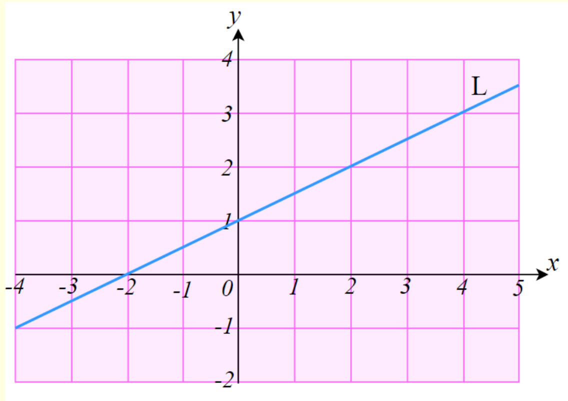
$y = mx + c$

Step 1: Find m

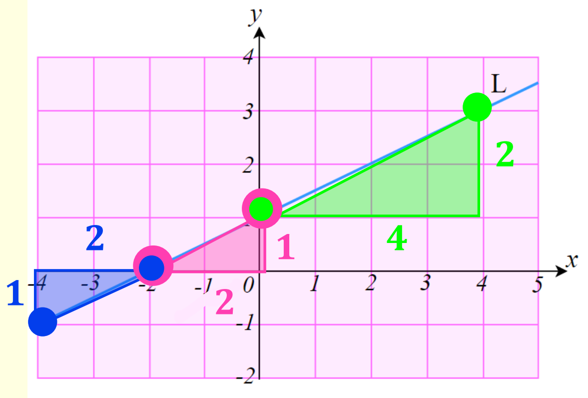
There are 4 ways to find this dependent on what we're given

Way 1: If given graph - pick any 2 points on the line, form a triangle & work out the $\frac{rise}{run}$

e.g. Find the slope of the following line below on the graph on the left



Solution $\Longrightarrow$
pick a pair of points and form any triangle (above or below the line) and work out the $\frac{rise}{run}$



It doesn't matter which triangle we build (all give the same answer -). Let's use all 2 triangles formed above.

$$\frac{\text{rise}}{\text{run}} = \frac{1}{2} \quad \text{or} \quad \frac{1}{2} \quad \text{or} \quad \frac{2}{4} = \frac{1}{2}$$

The slope is positive is the line is going up from left to right (rise) and negative if the line is going down from left to right, so we know that have a positive slope.

$$y = \frac{1}{2}x + c$$

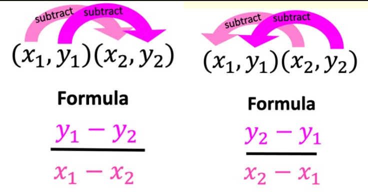
Alternative method: we can just write down any 2 points ("nice points" that are whole numbers) from the graph and proceed as in way 2 below

Way 2: If given 2 points - use the following slope formula:

e.g. Find the equation of the line passing through the points $(-1,3)$ and $(2,4)$

$$m = \frac{4-3}{2--1} = \frac{1}{3} \qquad \text{or} \qquad m = \frac{3-4}{-1-2} = \frac{-1}{-3} = \frac{1}{3}$$

$$y = \frac{1}{3}x + c$$

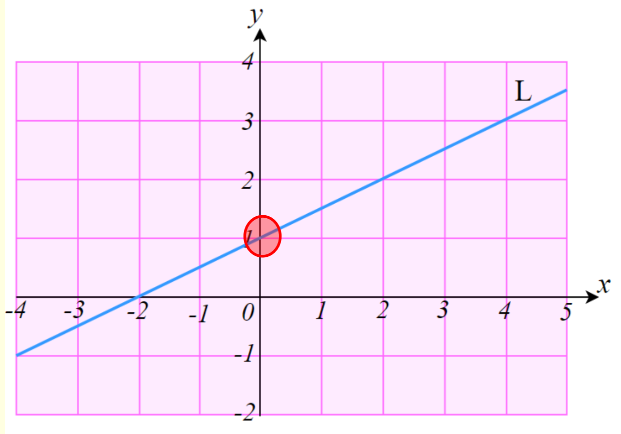


Step 2: Find c

There are 2 ways to find this dependent on what we're given

Way 1: If given the graph -
c is just the value where the graph crosses the y axis. We can read this off easily.

e.g. Find the y intercept of the following line



using step 1 we know we have $y = \frac{1}{2}x + c$

We can see that c is 1 from the graph (red circle)

$$y = \frac{1}{2}x + 1$$

Way 3: If given a line that parallel to – locate slope and use same slope

e.g. 1 Find the line parallel to $y = 2x - 3$

$y = 2x - 3$ has gradient 2. Since parallel means the same gradient, we use the same gradient 2.

$$y = 2x + c$$

e.g. 2 Find the line parallel to $6x + 2y = 5$

we must first re-arrange using algebra to get into the form $y = mx + c$. We do this in order to spot the gradient.

$$2y = -6x + 5$$
$$y = \frac{-6x + 5}{2}$$
$$y = -3x + \frac{5}{2}$$

$y = -3x + \frac{5}{2}$ has gradient -3 . Since parallel means the same gradient, we use the same gradient -3.

$$y = -3x + c$$

Way 4: If given a line perpendicular to – locate slope and use negative reciprocal slope (negative reciprocal means we flip the fraction and change the sign)

e.g. 1 Find the line perpendicular to $y = 2x - 3$

$y = 2x - 3$ has gradient 2 . Since perpendicular means the negative reciprocal gradient $-\frac{1}{2}$

$$y = -\frac{1}{2}x + c$$

e.g. 2 Find the line perpendicular to $4x + 2y = 6$

we must first re-arrange using algebra to get into the form $y = mx + c$. We do this in order to spot the gradient.

$$2y = -6x + 5$$
$$y = \frac{-6x + 5}{2}$$
$$y = -3x + \frac{5}{2}$$

$y = -3x + \frac{5}{2}$ has gradient -3. Since perpendicular means the negative reciprocal gradient, we use the negative reciprocal $\frac{1}{3}$

$$y = \frac{1}{3}x + c$$

Way 2: If given a point passes through - plug in the point since the point (x, y) tells us what x and y are. Then solve for c using algebra.

e.g. Find the line parallel to $y = 2x - 3$ and passing through $(-1,4)$

using step 1 (way 3) we know we have slope 2 hence $y = 2x + c$

Now we plug in the point $(-1\ ,\ 4)$ into $y = 2x + c$.
This means we replace x with -1 and y with 4 and solve for c

$$4 = 2(-1) + c$$

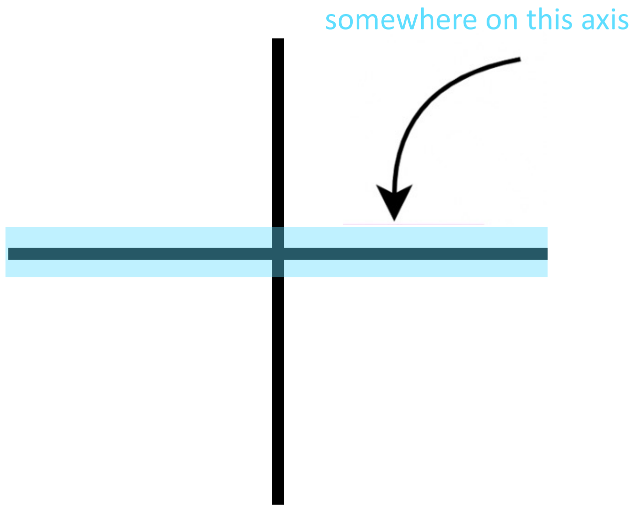
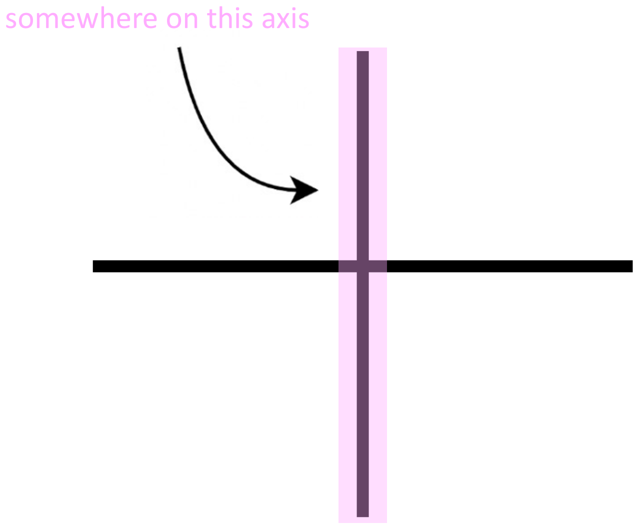
Solve for c using algebra

$$4 = -2 + c$$
$$c = 4 + 2 = 6$$
$$y = 2x + 6$$

Note:
If given 2 points that the line passes through, choose either point to plug in. Both will give the same answer for c.

How do we find x and y intercepts when given an equation in any form?

The x intercept is the point where the graph crosses the x axis and the y intercept is the point where the graph crosses the y axis

x intercept	***y intercept***
 To find this point we set $y = 0$ (we replace y with 0) and solve for x. The coordinate will be $(x, 0)$ where x is the value found.	 To find this point we set $x = 0$ (we replace x with 0) and solve for y. The coordinate will be $(0, y)$ where y is the value found. Remember that we can also just quickly read this value off from an equation. It is the c value.